Evaluation of Forecasting and Inventory Control in Multi-product Manufacturing Systems Operating under Erratic Demand: A Case Study in the Automotive Domain

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Abstract: Erratic demands have traditionally shown complexity in inventory management and forecasting. Several organisations, especially in multi-product manufacturing systems with intermittent demand high level of variability and uncertainty, assume suitable estimators are put in place to forecast demand. This forces them to pay attention to controlling the inventory of a system. The forecasted demand is often characterised by one or more (hybrid) statistical distributions. The latter, often used in modelling highly variable intermittent demand profile, has a significant effect on the inventory and service level in a system. This paper evaluates the demand distributions frequently used for inventory control under erratic demand variability. We use a discrete event simulation technique to analyse the level of significance of statistical distributions. We then prove our methodology on real industrial data of a multi-product manufacturing plant. We aim to determine the most suitable approach for inventory control with respect to minimising inventory while maintaining zero stock-outs and high service level.

Key words: Manufacturing systems, erratic demand, parametric forecasting, statistical distributions, simulation.

1. Introduction

Demand profiles characterised by uneven transactions of highly variable demand sizes are often referred as erratic demands. It is distinguished by intermittent operations and transactions, and occurs when demands are depicted with high variations and irregular sizes at different intervals. This intermittent demand pattern plays a significant effect in forecasting and inventory control of a system. This is particularly true in multi-product manufacturing environments. Demand distributions frequently used in modelling erratic demand patterns assume that demand profile conforms to a particular probability distribution. Most studies on inventory control selected a particular probability distribution based on the assumption of having an appropriate demand estimator and a lead time demand distribution [1]. According to Syntetos et al. [2] the main issues challenging this sector are (a) advance development of robust operational classifications of erratic demand for the purpose of forecasting and stock control, (b) appropriate modelling of the demand characteristics in order to suggest suitable or more powerful estimators for adequate stock control. These two concerns directly relate to the hypothesised distribution used for modelling the demand profiles. The paper showed that
these issues have not been addressed.

Furthermore, selection and implementation of suitable estimation techniques, appropriate stock control strategies and adequate demand distributional assumptions are often complex especially in multi-product manufacturing environments with erratic demand. This complexity is based on the issue of no clear classification and understanding of the effect and the relationship between operationalised SKU (stock keeping unit) classification methods and demand distributional assumptions. The uncertainty of demand variability requires adequate distribution. Several studies that used parametric techniques employed either single or compound distributions based on the determination of the coefficient of variation in order to determine the level of variability in a system [2-4]. Several application software packages have been designed to aid forecasting and inventory control in industries. Fildes et al. [1] showed a rapid development in forecasting and inventory control. Syntetos et al. [2] investigated effect of erratic demand on statistical distributions of spare parts with three data sets of 13,000 stock keeping units (SKU). Their findings suggested that further investigation is necessary with consideration to the goodness-of-fit of various distributions to the lead time demand for clearer understanding in this area. The main issue is the challenge to present a simple and appropriate framework for forecasting and inventory control under erratic demand conditions. Several studies have shown that forecasting techniques are significantly inaccurate especially in when a trend is observed in demand data [5-7]. Porras and Dekker [7] compares different re-order point techniques of a spare parts company via ex-ante and ex-post approaches for optimisation of the spare parts inventory control system. The paper observed that a majority of the empirical studies in the spare parts industry focused on forecasting techniques for demand of slow moving parts while few studies implement empirical techniques in inventory models such as re-order points and economic order quantity. Their findings showed that the empirical model outperformed Poisson and Willemain models in terms of a higher fill rate and cost reduction.

This paper investigates the behaviour of demand distributions in a multi-product manufacturing firm. The objective is to use an empirical technique to analyse various statistical distributions under erratic demand condition via the goodness-of-fit test and compare simulated results with theoretical probabilities. Experimental data of five products with erratic demand profiles were investigated using a simulation approach. The datasets used were collected from an automobile electronics component manufacturing company. The remainder of this paper is organised as follows: Section 2 presents the literature review of forecasting and demand distribution of erratic demands; Section 3 describes the research methodology and Section 4 provides the description of empirical technique, data and simulation results; Section 5 presents the discussion and conclusion of the study.

2. Literature Review

Organisations aim to meet customer demands in the most valuable means within a reasonable time frame. Customer demands are in most cases irregular in sizes and infrequent. This section focuses on a brief review of the forecasting and demand distributions of erratic demands.

2.1 Forecasting

Forecasting estimates the proportion of a part or service that will be needed. In most cases it uses quantitative methods, for instance, historical sales data from the research market in making various decisions; such as demand volume and price rise or fall of products. Several organisations use forecasting to put an appropriate stock in place in order to: (1) respond swiftly to actual demand and (2) to reduce their inventory. Predicting future demand with high
accuracy to represent the actual demand is naturally difficult [8]. Several forecasting techniques have been developed over the years with various innovations in order to forecast demand with high accuracy. These forecasting techniques are based on the principles of constant and/or trend model. The constant model represents demands that occurred in different periods as independent random deviations [9]. Constant model is suitable for regular moving products with no seasonal pattern. Conversely, trend model considers linear trends in demand such that it represents demand as a combination of the demand trend and constant model.

Some of the widely used forecasting techniques include: the SMA (simple moving average), SES (single exponential smoothing), DES (double exponential smoothing), Croston, and SBA (Syntetos Boylan approximation). SMA utilises constant model in demand forecasting, such that the average of the most recent periods of the observed data is taken into consideration. Similarly, SES uses constant model, however it requires a linear combination of the previous forecast and the recently gathered demand data to update the forecast in a given period. DES uses trend model such that an expectation of slow increase or decrease in demand over a period is established from the observed data [10]. Croston technique implements SES (it uses constant model) and in cases where the time between two successive demand data is large enough, the adjustment of the techniques parameters is recommended in order to handle the variations. Likewise SBA is based on the concept of Croston with less bias and improvement in its variation adjustments [6]. These techniques smooth demand data, easily spot trends and remove short term noises from demand datasets. However, several drawbacks found in their implementations form the bases for additional search for clearer understanding, improvement and advancement in this area.

SMA allows a substantial proportion of the signal that is shorter than its window length resulting in unexpected products appearing where they are not expected in the data. This arises when a higher rate of occurrences are not properly removed. SES has similar drawbacks to that of SMA, such as allowing a lag comparative to input data [11]. Additionally, SES has significant errors when there is a trend in the data [12]. DES is often used when a trend is observed in the data. However, the time needed to determine an appropriate weighting factor alpha (α) and the method of continuous monitoring of α for adequate updating α adversely influences the performance of DES. Croston technique lags when trends are present in a data and shows bias when the time between two successive data has high variations [5]. SBA has been shown to outperform Croston yet it requires further testing especially with erratic demand profile to have a better understanding of its behaviour and relationship with complex data.

To model demand profiles using any of the forecasting techniques, adequate knowledge of demand distribution is required. Accuracy in forecasting and inventory control is dependent on the estimation of demand distribution parameters. However, it is this estimated demand distribution parameters that determine the decision parameters, for instance the ROP (re-order point) of inventory of a system. Distinct variables are used for different inventory system, in order to determine appropriate demand distributional parameters for a system. For instance the period inventory control strategy (T, s, S) widely studied, has no simple framework or algorithms for determining the demand distribution parameters. The use of various known variables in generating data is often supported by various heuristic procedures [13]. Other strategies that utilised various known variables depending on the inventory system under study include; the power approximation, Naddor’s heuristic and the normal approximation [14-17]. Estimating especially mean and variance of demand in parametric forecasting is vital and various assumptions and/or hypothesis are made for
simplification purpose of the forecasting procedure in order to estimate demand distribution parameters.

2.2 Demand Distributions

Demand profile varies based on their characteristic variability. The four main classifications of demand profile via the level of their variability in the literature are (1) slow-moving, (2) intermittent, (3) erratic and (4) lumpy demand profiles [5, 6, 8, 18, 19]. The slow-moving demand profile has irregular demands with individual demand size equivalent to one or few demands. The intermittent demand profile has random demand with no demand occurring in some intervals. The erratic demand profile has irregular demand sizes with a high level of variations while lumpy demand profile is characterised by random demand with zero demand in some intervals and demand sizes with a high level of variations. The reader is encouraged to check Syntetos and Boylan [6] for additional information on the classification of demand profiles. The following discussion is of interest to erratic profiles. The latter is the main focus of this research.

Erratic demand profile is characterised by irregular demand sizes with high variations. The probability of measuring uncertainty is a critical issue in quality control and manufacturing system performance [20]. The uncertainty of demand variability found in erratic demand adversely affects the performance of a manufacturing system because there has not been any appropriate mathematical model to accurately predict the system. The demand sizes and intervals are often used for modelling of demand profiles in parametric modelling. The demand parameters such as the demand variation and the mean of the demand profile often determined using the MSEs (mean squared errors) are fitted to statistical distributions. Some of the statistical distributions in certain conditions outperform their alternatives. According to the findings of Porras and Dekker [7], normal distribution outperforms Poisson distribution based on the level of sensitivity to changes in fill rate. Normal distribution is less sensitive to fill rate than Poisson distribution. Syntetos et al. [2] suggested a Bernoulli process from the geometric distribution that may be used to generate demand when time is as a discrete variable. However, if time is regarded as a continuous variable, Poisson would generate negative exponentially distributed time between demands or inter-arrival intervals. Studies like Kwan [21] and Janssen [22] agree with the findings that geometric and exponential distributions are useful for representing the time between demands.

Issues such as failure of a distribution to properly model the demand size and interval led to development of various compound distributions. The issues of arbitrary distribution of demand sizes in Poisson distribution led to the development of a combination of Poisson distribution and geometric distribution referred as stuttering Poisson. Stuttering Poisson is found useful for demand occurrence and demand size [2, 3, 23]. Additionally, Vereecke and Verstraeten [4] suggested that a combination of Poisson distribution and normal distribution for demand occurrence and demand sizes is useful in modelling erratic demand conditions. However, normal distribution in uneven demands skews towards the right making the normality assumption inapplicable. According to Quenouille [24], Poisson-Logarithmic distribution produces NBD (negative binomial distribution). Total demand is noticed to be NBD, when time between demands is assumed to be Poisson distributed and the demand size is uneven, nonetheless trails a logarithmic distribution.

Erratic demands are further represented by gamma distribution. This distribution is a continuous analogue of NBD and it covers a wide range of distribution shapes. It is useful especially for non-negative values. It gets a good mathematical tractability in inventory control [2, 25-28]. However, in cases where demand is assumed to be discrete, gamma distribution is
approximated to the distribution of demand. This implies that mean and variance of demand are estimated in cases of NBD and gamma distribution [2]. Kwan [21] suggested a compound distribution referred as LZP (log-zero-Poisson). LZP occurs when time between demands follows a Bernoulli process and demand sizes follows Logarithmic-Poisson distribution. LZP has three parameters and it uses a complex estimation technique.

Croston [29, 30] suggested normal distribution which has negative values when demand skews to the left. When it skews to the right due to uneven demand, the normality assumption becomes infeasible. Syntetos et al. [2] suggested that the normal distribution may be useful for lead time demand, when lead times are long. The paper stated that the normality assumption is feasible because long lead times allow central limit theorem effects for the total demands over a corresponding period. Another reason for the usefulness of normality assumption is when the CV (coefficient of variation) of demand distribution over a period is small. The third reason for acceptability and wide use of normality assumption is because its algorithms are simple to apply and execute.

Specific demand distribution is proposed in the literature for erratic demand depending on the demand distribution parameters. Most of the demand distributions incorporate compound distributions determined from distributions of the demand intervals and demand sizes. Over the years, the use of compound Poisson distribution for demand per unit time while restricting the lead times to constant has been widely used in the literature. However, the proposal by Croston [29, 30] presented the appropriateness of normal distribution for erratic demand. Croston’s [29, 30] findings showed that normal distribution is superior to exponential smoothing when specific distributions are assumed for demand arrival and demand size.

The practicability issues regarding the probability models for lead time demand for most of the researched distributions for erratic demand in the multi-product manufacturing environment have not been adequately considered. Mitchell et al. [31] suggested the appropriateness of compound Poisson distributions for demands in a case of the United States Air Force. The issue of non-validation of distribution using real-industrial data of most researches in this area could be attributed to non-availability of real-data for researches. The need for validation of selected distributions for erratic demand using a large quantity of real-data would provide a good contribution to the practicability of such distributions in erratic demand environment.

Contrary to the suggestion of superior performance of Croston’s technique, the outcome of data collected from industries showed poor performance in terms of inventory control, when compared with simulation results [2, 5, 6]. In order to substantiate the superiority of normal distribution over other distributions, further investigation using a large quantity of real-industrial data would be needed to provide a useful comparison.

Poisson and compound Poisson have been widely studied in forecasting and inventory control of erratic demand. However, in practice, most of the erratic demand profiles fit appropriately to continuous probability distribution. In this paper, a study of continuous probability distributions based on the real-industrial data especially on normal, triangular and beta distributions is carried out. Analysis of comparison of their performance is provided in the following section.

3. Methodology

In this section, a time series model was developed to investigate the behaviour of various forecasting techniques when observations have high variations, and, the variations are uncertain. Time series model was adopted in this study because it describes the demand datasets collected which are random with fixed demand intervals. In time series modelling, the
demands are assumed to be probabilistically distributed via a time function. While other modelling approaches exist such as Delphi approach, the time series approach is appropriate to demand profiles with automatic, short-term forecasting of the fixed demand time of the dataset [32-36].

3.1 Mathematical Definitions of a Demand Profile

The demand dataset used in this study represents a monthly demand for multi-products. The demand observations are represented hereinafter as $x_t$, while subscript $t$ denotes the time period. Therefore $x_t$ indicates known observations obtained in a month’s period, with 12 months demand data in a cycle. A graphical representation of a time series of one of the five demand dataset examined here is shown in Fig. 1.

Fig. 1 is used to determine a model with adequate description of the observed data and permits high accuracy in estimation of future occurrence. The time series in this case has a constant element with value $b$, having variations about $b$, which is determined by a random variable denoted as $\varepsilon_t$. The random variation $\varepsilon_t$ about the mean value is referred to as noise ($E[\varepsilon_t]$) and is assumed to be zero at its mean value and has a give variance ($\text{Var}[\varepsilon_t]$). Mathematically, a time model from Fig. 1 is developed as follows:

An unknown demand ($X_t$) in time period $t$ is given by

$$ X_t = b + \varepsilon_t $$

(1)

The noise variation in two different time periods $E[\varepsilon_t \varepsilon_w]$ is assumed to be independent and has zero value, such that

$$ E[\varepsilon_t] = 0, \text{Var}[\varepsilon_t] = \sigma^2 $$

$$ E[\varepsilon_t \varepsilon_w] = 0 \text{ for } t \neq w $$

where $\sigma$ is the standard deviation of noise.

If the observation shows a linear trend $b_1$, then $X_t$ is given by

$$ X_t = b_0 + b_1 t + \varepsilon_t $$

(2)

Eqs. (1) and (2) are distinctive types of a polynomial model, given as

$$ X_t = b_0 + b_1 t + b_2 t^2 + \ldots + b_n t^n + \varepsilon_t $$

If the observation shows a seasonal trend for instance four seasonal variations in a year, then $X_t$ is given by

$$ X_t = b_0 + b_1 \sin \frac{2\pi t}{4} + b_1 \cos \frac{2\pi t}{4} + \varepsilon_t $$

From all the mathematical models developed so far, it is obvious that the time series is a function of time and the model parameters. Therefore, $X_t$ could be written as

$$ X_t = f(b_0, b_1, \ldots, b_n, t) + \varepsilon_t $$

The value of $f$ is constant at any given time $t$ and the assumed value of $\varepsilon_t$ is zero, therefore,

$$ E[X_t] = f(b_0, b_1, \ldots, b_n, t), \text{ while variance of} $$

$$ X_t (\text{Var}[X_t]) = \text{Var}[\varepsilon_t] = \sigma^2. $$

Fig. 1  Time series demand data.
The two important variable elements of a time series model are the mean which varies with time and the difference from the mean, which varies randomly.

3.2 Fitting Model Parameters

Identification and fitting of suitable model parameters are important because an optimal setting for smoothing parameters reduces the forecasting inaccuracy from Theil’s U statistic range rules [37]. Recent observations are often considered than old observations, as data are likely to change with time. This could impact the outcome when such changes are significant and are not accounted for by the parameters. Therefore, it is required for a model estimate to account for changes in demand data.

Assuming the estimated model parameters has a bar over the variable to indicate that it is estimated, for instance the time series parameter \( \tilde{\alpha} \) is given as

\[ \tilde{\alpha}, \tilde{\beta}, ... \tilde{\beta}_n \]

Similarly, estimated standard deviation is given as \( \tilde{\sigma} \).

For simple constant model as describe in Eq. (1),

\[ \tilde{\beta} = \sum_{t=1}^{n} x_t, \text{ where } n \text{ is the sum total of the observed data} \]

For moving average model, \( \tilde{\beta} = \sum_{t=m+1}^{n} \left( \frac{x_t}{m} \right), k = t - m + 1, \text{ where } m \text{ is the average of the last sampled data} \]

The constant and moving average models will yield large errors in presence of trend with high variations because they have constant anticipated values over a period of time and are not suitable for the erratic demand profiles used in this study [32-36]. Similarly, the single exponential smoothing for the constant model assumes that the time series is a constant model, such that \( \tilde{b} \) is assumed as the weighted average of the latest observed data and the latest estimate, which is given as

\[ \tilde{b}_T = \tilde{b}_{T-1} + \alpha x_T + (1 - \alpha) \tilde{b}_{T-1} \]

where \( T \) is the current time, \( \alpha \) is a parameter with interval between (0 and 1).

Single exponential smoothing has lagging characteristics that is similar to the moving average leading to a high level of error in forecasting demand with trend and uncertainty. It is therefore not suitable for erratic demand profiles [32-36]. It is essential to include trend element in the time series model in order to address the problem of trend in forecasting techniques used.

Therefore, Eq. (2) becomes

\[ X_T = \alpha_T + b_T (t - T) + \epsilon_t \quad (3) \]

where \( a \) accounts for constant element and \( b \) accounts for linear trend element of the model.

Regression model uses the latest \( m \) observations and the slope in estimating the smoothing parameters and is given by

\[ \tilde{a}_T = \frac{6}{m(m+1)} S_2(T) + \frac{2(m-1)}{m(m+1)} S_1(T) \text{ and } \tilde{b}_T = \frac{12}{m(m^2-1)} S_2(T) + \frac{6}{m(m+1)} S_1(T) \]

where \( S_1 \) and \( S_2 \) are the sums of the latest \( m \) observations and are given as

\[ S_1(t) = \sum_{k=t-m+1}^{t} x_k \text{ and } S_2(t) = \sum_{k=t-m+1}^{t} (k - t) x_k \]

\[ X_{T+t} = \tilde{a}_T + \tilde{b}_T, \text{ for all } t > 0 \]

where \( t \) is an increase in time from current time \( T \).

\[ X_{T+t} = \frac{6}{m(m+1)} S_2(T) + \frac{2(m-1)}{m(m+1)} S_1(T) + \frac{12}{m(m^2-1)} S_2(T) + \frac{6}{m(m+1)} S_1(T) \]

The regression model responds to trend when the noise from the time series is assumed to be zero. There is lagging when the noise variance is greater than zero, leading to under and/or over correctness as the slope changes owing to its method of adjusting to the changing mean value [32-34]. The lagging effect in regression model makes it unsuitable for observations with uncertainty in demand variability.

Double Exponential Smoothing technique takes into consideration the constant element and the linear trends element using two parameters \( \alpha \) and \( \beta \), such that from Eq. (3), \( \tilde{a}_T \) and \( \tilde{b}_T \) are given by
\[ \tilde{a}_T = \alpha x_T + (1 - \alpha)(\tilde{a}_{T-1} + \tilde{b}_{T-1}) \]
\[ \tilde{b}_T = \beta(\tilde{a}_T - \tilde{a}_{T-1}) + (1 - \beta)\tilde{b}_{T-1} \]
\[ X_{T+r} = \tilde{a}_T + \tau\tilde{b}_T, \text{ for all } r > 0, \]
\[ X_{T+r} = \alpha x_T + (1 - \alpha)(\tilde{a}_{T-1} + \tilde{b}_{T-1}) + \tau(\beta(\tilde{a}_T - \tilde{a}_{T-1}) + (1 - \beta)\tilde{b}_{T-1}) \]

In the presence of variations, the starting and ending stages of forecasting using the Double Exponential Smoothing technique has under and overshoot. However, when variations decrease the model begins to track the mean value. Reaction to trend is slow as it keeps track of the past observations resulting to slow recovery from perturbation. A review of several literatures shows that there is no single framework for selection of a suitable forecasting technique that gives accurate forecast especially when the demand is erratic [32-36]. Individual forecasting technique has its merits and drawbacks, therefore it is necessary to construct a quantitative forecasting technique that accounts for demand data dynamics, such that the relationship between the variables can be estimated using the time series analysis.

### 3.3 Nonlinear Constraints of Manufacturing Systems

Linear time series models are typically used in modelling and controlling of the process of material flow in a manufacturing system. However, such models disregard modelling definite features of a manufacturing system. Erratic demand is unpredictable and it is practical to adopt a suitable time series model that effectively represents the non-linearity of the data in order to adequately coordinate the planning of such a system. In linear modelling, parameters such as the work-in-process inventory, backorder, linear material flow balance equations and variable lead time requirements are considered [38]. However, parameters such as the probability of stock-out control, to enable just in time delivery often referred to as QoS (quality of service), and the probability of maximum stock-out constraints, are not explicitly represented in linear models. QoS and lead time are significant nonlinear constraints in the manufacturing planning optimisation procedure of manufacturing systems. They require being appropriately modelled. In order to capture the statistics of the stochastic performance, of a manufacturing system shown during a short period of time, the non-linear constraints such as the lead time and QoS must be quantified. Inappropriate quantification of these non-linear constraints, due to the complexity in computation, has an adverse effect on the performance of a manufacturing system in terms of work-in-process control, just-in-time delivery and service level.

To examine the behaviour and effect of these non-linear constraints in a multi-product manufacturing system, the observed data were test using an empirical technique, which is one feasible option as it describes the relationship between aggregated quantities in observed data. This paper applied the empirical technique to study the relationship between variables of demand data via Microsoft Excel spreadsheet, which has been widely used in developing models for the purpose of forecasting [39, 40]. The demand dataset was grouped into two groups the first group was used to identify the relevant parameters and set up the technique for the experiments. The second group was used to perform forecast, and examine the accuracy of the techniques. To investigate the performance of the techniques, discrete event simulation approach was adopted in order to develop sets of non-dominated solutions for the experiments. The performance of the manufacturing system was evaluated via the inventory and service levels were examined.

### 4. Empirical Technique

An empirical technique was used to model the demand profiles. This will determine the inventory control parameters and, will assess the performance of statistical distributions under erratic demand profile, rather than the forecasting techniques. Empirical distribution samples demand profiles via past
demands, without issues arising due to demand irregularities and variations.

The empirical technique applied in this paper uses a constant lead time. To design an empirical model, a histogram of the demand profile with constant lead time is constructed without selection, such that demand data are grouped into a fixed interval. The process of constructing demand histogram without sampling individual demand data allows this technique to captures relationships. It also serves as a fixed demand interval that supports the scheduling of preventive maintenance. Empirical techniques have been shown in the literature to outperform forecasting techniques [2, 7, 41].

4.1 Experimental Data and Structure

The empirical data in this study is taken from a multi-product automotive electronics component manufacturing company with five demand profiles of automotive parts. The data covers a period of 37 months from May 2008 to May 2011. The Demand size is defined as the sizes of the actual demand. The time frame is 1 month of a total history length is 37 months and the actual lead time is 8 weeks. Table 1 and Fig. 2 provide detailed descriptive statistics of the data.

4.2 Data Descriptions and Analysis

In order to determine a good distributional model for the demand data, histogram of each demand profile was created. Fig. 3 shows histograms of the demand profile with Part-type 1, 2 and 4 partly or wholly representing a symmetric distribution, while Part-type 3 and 5 are skewed (non-symmetric) to the left. Symmetric distribution is one that has approximately two equal halves. For symmetric distribution the mean and standard deviation of the distribution are often used in modelling such distribution. However for skewed distribution, it is difficult to ascertain the centre of the distribution.

In most cases, the mean, mode, median, minimum and/or maximum values are used in skewed distributions. According to Anscombe [42], to model a distribution one needs to compute the sample mean, median and mode of the data and then, determine the best fit distribution. Statistical software known as

<table>
<thead>
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<th>Demand date</th>
<th>Part-type 1</th>
<th>Part-type 2</th>
<th>Part-type 3</th>
<th>Part-type 4</th>
<th>Part-type 5</th>
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<tr>
<td></td>
<td>Demand interval</td>
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<td>Demand size</td>
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<tr>
<td>May 2008</td>
<td>0 0 0 0 0 0 2 39420</td>
<td>0 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aug. 2010</td>
<td>4 9312 5 62600 1 1972 5 89928</td>
<td>5 29750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sept. 2010</td>
<td>4 10304 4 31320 5 13108 4 88848</td>
<td>4 25750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oct. 2010</td>
<td>5 10432 4 37720 4 11712 4 65448</td>
<td>5 36750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov. 2010</td>
<td>4 12000 5 67080 4 12192 5 62495</td>
<td>5 40000</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Dec. 2010</td>
<td>5 11168 4 65360 4 7680 4 41220</td>
<td>4 22250</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Jan. 2011</td>
<td>5 14464 4 62600 4 12672 5 65790</td>
<td>5 35500</td>
<td></td>
<td></td>
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<tr>
<td>May. 2011</td>
<td>1 3328 2 30960 1 3072 3 53590</td>
<td>4 28500</td>
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</table>
StatFit by Geer Mountain Software Corporation was used to determine the parameters of interest and, the auto-fit function was used to determine the best fit distribution. Fig. 3 shows the skewness of the demand data (symmetric or non-symmetric) and Table 2 provides detailed information on the parameters for modelling the distribution.

### 4.3 Goodness of Fit Test

The commonly used testing techniques for significance of fit of statistical distributions are the Chi-Square test and the K-S (Kolmogorov-Smirnov) test. They are useful in determining the degree of fit between empirical and anticipated frequencies. However, Chi-Square test requires grouping of the data into categories. The issue of grouping data into categories to enable each category has an anticipated frequency of no less than a minimum number of observations led to modifications of Chi-Square test [2]. The requirement to specify boundaries for a certain number of categories in the case of Chi-Square test for erratic demands makes it complex in use. The issue of grouping data into categories is not required in K-S test, which removes the difficulty of specifying the boundaries of grouped data. K-S test measures the empirical cumulative distribution function for a variable with identified hypothetical distribution, based on the assumption that the parameters of the distributions are identified in advance. One could argue that the power of K-S to detect deviations from the theoretical distribution is diminished owing to the issue of estimation of the critical values of a distribution.

The measurement of the entire distribution is unreliable and the measurement of the entire distribution could lead to a low demand value to cause poor forecast of high demand value. Syntetos et al. [2] agrees that in measuring the fitness of a distribution, attention should be given to the upper end of such distribution. As a result of this, we choose K-S test and a modification of K-S known as the Anderson-Darling (A-D) test. A-D considers the tails of a distribution more than the K-S test. Unlike K-S, it uses specific distributions in determining critical values. This is beneficial as it allows a more sensitive test and also eliminates the issue that critical values
Fig. 3 Relative frequency of demand size.
must be determined for each distribution [43, 44].

Four continuous probability distributions (Beta, Normal, Gamma and Triangular) were considered for the goodness of fit test based on the demand profile of five part-types. 1% and 5% level of significance based on K-S statistical tables was used to compute the critical values. The degree of fitness of distributions is considered as “Strong Fit”, if the p-value is smaller than critical values for 1% and 5% level of significance, “Good Fit”, if the p-value is smaller than the critical value for 1% but larger than 5% level of significance. If the p-value is larger than the critical values of 1% and 5% respectively, it is considered as “No Fit”. The summary of the goodness of fit test is presented in Table 3.

4.4 Demands during Lead Time

Erratic demand profiles are difficult to manage within a short lead time. Organisations often assume a continuous review policy in order to control inventory

Table 2  Demand descriptive statistics.

<table>
<thead>
<tr>
<th>Part-type</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Median</th>
<th>Mode</th>
<th>Standard deviation</th>
<th>Variance</th>
<th>Coefficient of variation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>96.</td>
<td>3904.</td>
<td>2049.9</td>
<td>2048.</td>
<td>2016.0</td>
<td>934.03</td>
<td>0.872e+006</td>
<td>45.5649</td>
<td>-0.1584</td>
<td>-0.6167</td>
</tr>
<tr>
<td>2</td>
<td>960.</td>
<td>25000.</td>
<td>12369.8</td>
<td>13620.</td>
<td>16520.0</td>
<td>5572.19</td>
<td>31.049e+006</td>
<td>45.0469</td>
<td>-0.3623</td>
<td>-0.7239</td>
</tr>
<tr>
<td>3</td>
<td>576.</td>
<td>3744.</td>
<td>2548.9</td>
<td>2880.</td>
<td>2880.0</td>
<td>878.94</td>
<td>0.773 e+006</td>
<td>34.4821</td>
<td>-0.5985</td>
<td>-0.9126</td>
</tr>
<tr>
<td>4</td>
<td>630.</td>
<td>28170.</td>
<td>16352.5</td>
<td>15895.</td>
<td>15857.5</td>
<td>5603.05</td>
<td>31.394 e+006</td>
<td>34.2643</td>
<td>-0.2145</td>
<td>-0.5985</td>
</tr>
<tr>
<td>5</td>
<td>1500.</td>
<td>8000.</td>
<td>6339.3</td>
<td>6500.</td>
<td>8000.0</td>
<td>6339.3</td>
<td>3.560e+006</td>
<td>29.7638</td>
<td>-1.0841</td>
<td>8.7172e-003</td>
</tr>
</tbody>
</table>

Table 3  Goodness of fit test results (maximum likelihood estimates, level of significance = 0.01 and 0.05, accuracy of fit = 0.0003).

<table>
<thead>
<tr>
<th>Demand data</th>
<th>Distribution</th>
<th>Kolmogorov Smirnov</th>
<th>Anderson Darling</th>
<th>Degree fitness (strong, good or no fit)</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part-type 1</td>
<td>Beta</td>
<td>0.142</td>
<td>2.14</td>
<td>Good fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.074</td>
<td>0.37</td>
<td>Good fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>0.132</td>
<td>1.44</td>
<td>Good fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td></td>
<td>Triangular</td>
<td>0.268</td>
<td>9.21</td>
<td>No fit</td>
<td>Reject</td>
</tr>
<tr>
<td>Part-type 2</td>
<td>Beta</td>
<td>0.123</td>
<td>0.46</td>
<td>Good fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>No fit</td>
<td>No fit</td>
<td>No fit</td>
<td>Reject</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>0.177</td>
<td>1.10</td>
<td>Good fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td></td>
<td>Triangular</td>
<td>0.139</td>
<td>1.21</td>
<td>Good fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td>Part-type 3</td>
<td>Beta</td>
<td>0.205</td>
<td>35.80</td>
<td>No fit</td>
<td>Reject</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>No fit</td>
<td>No fit</td>
<td>No fit</td>
<td>Reject</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>0.060</td>
<td>0.48</td>
<td>Strong fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td></td>
<td>Triangular</td>
<td>0.110</td>
<td>1.80</td>
<td>No fit</td>
<td>Reject</td>
</tr>
<tr>
<td>Part-type 4</td>
<td>Beta</td>
<td>0.221</td>
<td>3.43</td>
<td>No fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>No fit</td>
<td>No fit</td>
<td>No fit</td>
<td>Reject</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>0.223</td>
<td>3.50</td>
<td>No fit</td>
<td>Do not reject</td>
</tr>
<tr>
<td></td>
<td>Triangular</td>
<td>0.406</td>
<td>35.41</td>
<td>No fit</td>
<td>Reject</td>
</tr>
</tbody>
</table>
and avoid stock-outs especially in an environment under intermittent demand profile. However, lead times often determine if stock-outs would occur. Continuous monitoring of inventory allows the adjustment of the time for replenishing order; however stock-outs occurrence depends on the lead time and level of demand variability. If demand is high, inventory reaches the ROP (re-order point) quickly, leading to a quick replenishment order. If demand is low, inventory drops slowly to the ROP, leading to a delayed replenishment order. However, once a replenishment order has been placed, the available safety inventory \(ss\) covers for uncertainty of demand during this period depending of the lead time and the level of demand. It is worthwhile to highlight the economic effect of quick replenishment orders. They add up to a heavy financial burden as, most probably, un-economic means of transportation such as air freight will be used.

Using the traditional approach in inventory management, the assumption of normality is implemented. The mean \(\mu\) and standard deviation \(SD\) of normal distribution are \(\mu = 2,050\) and \(SD = 934\), respectively which is represented in this paper as \(\sim N(2,050, 934)\), is used for the generation of inventory parameters. The corresponding inventory parameters of each distribution were generated using the same CSL and lead time \(L\) to allow for supplier planning for all distributions.

Based on the remarks obtained from the Goodness of Fit results in Table 3, the two best fitted distributions were selected for comparison, where the remark has only one “Do Not Reject” as in the case of part-type 4, approximated normal distribution was used to compare with the selected distribution. The performance of the distributions was compared to the actual demand during lead time distributions. Tables 4 and 5 highlight the difference between the actual fitted distribution and the normality assumption. The outcome of the normal distribution showed stock-outs. However, under the same CSL other distributions performed better than the normal distribution with respect to stock-outs.

- **Mathematical formula and notations:**
  - \(CSL = \text{Prob} (\text{Demand during lead} \leq \text{ROP})\)
  - \(\text{Mean demand during Lead: } D_L = D \times L\)
  - Safety stock: \(ss = \text{ROP} – D_L\) \hspace{1cm} (4)
  - \(\sigma_L = \sqrt{L} \times \sigma_D\)
  
  where \(\sigma_D\) is the standard deviation of demand per period (forecast error) \(\sigma_D = 1.25 \times MAD\) (mean adjusted deviation).

  - **Probability density function**
    \[ f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{(x - \mu)^2}{2\sigma^2} \right] \]

  - **Cumulative distribution function**
    \[ F(x, \mu, \sigma) = \int_{x=\infty}^{x} f(x, \mu, \sigma) \, dx \]

  \[ CSL = F(\text{ROP}, D_L, \sigma_L) \]

  \[ CSL = F(D_L + ss, D_L, \sigma_L) \]

  \[ ss = F^{-1}(CSL, D_L, \sigma_L) \]

  \[ ROP = D_L + ss = \]

  \[ \min \frac{a + \sqrt{CSL(b-a)(c-a)}}{b - \sqrt{(1-CSL)(b-a)(b-c)}} \quad a < ROP < c \] \hspace{1cm} (6)

  \[ \mu = \frac{a+b+c}{3} \] \hspace{1cm} (7)

- **Minimum Inventory at zero Stock-outs**
  \[ 0.4 \leq CSL \leq 1 \]

### 4.5 Cycle Service Level

CSL (cycle service level) is described as the probability of not having stock-outs during a cycle, or the fraction of replenishment cycles that end with all demand met. Assuming that demand across periods is independent (not correlated to a certain extent) and demand during lead time is normally distributed. CSL impacts the trade-off between inventory and stock-outs (assuming the appropriate distribution has been identified). Optimal CSL levels along with the rest of inventory parameters are determined for every part-type. 80% CSL often widely used in the literature
Table 4  Lead time result of distribution parameters for part-types 1-3.

<table>
<thead>
<tr>
<th>Part-type 1</th>
<th>Part-type 2</th>
<th>Part-type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSL = 80%</td>
<td>CSL = 80%</td>
<td>CSL = 67%</td>
</tr>
<tr>
<td>Distribution</td>
<td>Distribution</td>
<td>Distribution</td>
</tr>
<tr>
<td>Triangular</td>
<td>Triangular</td>
<td>Beta</td>
</tr>
<tr>
<td>$a = 768$</td>
<td>$\mu = 2050$</td>
<td>$\mu = 12370$</td>
</tr>
<tr>
<td>$c = 21680$</td>
<td>$SD = 934$</td>
<td>$SD = 5572$</td>
</tr>
<tr>
<td>$b = 31200$</td>
<td>$D_L = 8 \times \mu$</td>
<td></td>
</tr>
<tr>
<td>$\mu = 2235.3$</td>
<td></td>
<td>$\mu = 2552$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mu = 2549$</td>
</tr>
</tbody>
</table>

Demand during lead time $D_L = 17883$ $D_L = 16400$ $D_L = 98960$ $D_L = 20416$ $D_L = 20392$

$\sigma_L = 2641.8$ $\sigma_L = 15760$ $\sigma_L = 2486$

Inventory parameter $ss = 5706$ $ss = 2223$ $ss = 35412$ $ss = 13264$ $ss = 4767$ $ss = 1094$

ROP = 23588 $ROP = 18623$ $ROP = 146505$ $ROP = 112224$ $ROP = 25183$ $ROP = 21486$

Stock-outs 0 2328 0 160744 0 262

Table 5  Lead time result of distribution parameters for part-types 4 and 5.

<table>
<thead>
<tr>
<th>Part-type 4</th>
<th>Part-type 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSL = 98.55%</td>
<td>CSL = 46.05%</td>
</tr>
<tr>
<td>$\mu = 16400$</td>
<td>$\mu = 16352$</td>
</tr>
<tr>
<td>SD = 5580</td>
<td>SD = 5603</td>
</tr>
</tbody>
</table>

Distribution | Approximated normal | Normal | Beta | Normal |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_L = 131200$</td>
<td>$D_L = 130816$</td>
<td>$D_L = 50721$</td>
<td>$D_L = 50712$</td>
</tr>
<tr>
<td>$\sigma_L = 15783$</td>
<td>$\sigma_L = 15848$</td>
<td>$\sigma_L = 5337$</td>
<td>$\sigma_L = 5337$</td>
<td>$\sigma_L = 5337$</td>
</tr>
</tbody>
</table>

Demand during lead time

Inventory parameter $ss = 34482$ $ss = 34624$ $ss = 5029$ $ss = 0$

ROP = 165682 $ROP = 165440$ $ROP = 55750$ $ROP = 50712$

Stock-outs 0 4036 0 74590 0 74590

was used in for instance in part type 1, which resulted in high inventory. However, a better approach was to determine the optimal CSL that will fulfill our target goal (which is to minimize inventories while maintaining zero stock-outs). Optimal CSL was generated and used for the part-types. For instance, part-type 1 with optimal CSL of 0.512 at zero stock-outs showed a reduction in inventory by 34%. ROP = 18821 and $ss = 939$. The result of the optimal CSL while maintaining zero stock-outs is presented in Table 6. In the case of approximated normal distribution in part-type 4, the optimal CSL of over 98% showed a higher ROP than normal, implying that the approximated normal distribution was misleading.

4.6 Simulation of Inventory Parameters

A simulation model was developed in order to compare the analytical calculation and the assumptions. The objectives to ensure adequate inventory control while maintaining zero stock-outs in the system. The ROP was optimised to minimize inventory while maintaining zero stock-outs. Arena 13 from Rockwell systems was used for the simulation. One thousand orders were generated from the fitted distribution of a part-type and the calculated ROP of a part-type were tested. 95% confidence interval with 20
replications for each simulation run was considered adequate. The result of the experiment is shown in Table 7. The simulation outcome shows that all part-type has stock-outs. On the other hand, the parameters obtained using simulation led to almost zero stock-outs.

5. Discussion and Conclusions

In this section, we analysed the findings from the different experiments and presented the conclusion of the study. The results of the study show that the normality assumption would lead to stock-outs when the demand is non-linear. Furthermore, better results were obtained when the correct \( D_L \) distribution was used instead of the normal approximation. It is also worth pointing out to the demand of part-type 4, where we showed that even though the demand follows a normal distribution, the approximated normal distribution performed better with zero stock-out. The simulated ROPs in Table 7 represented a better option to use for production as they guaranteed almost zero stock-outs. Good inventory parameters are obtained when the correct \( D_L \) distribution is used. We showed that using \( D_L \) into its distribution is definitely a better option than using the normal approximation, stock-outs would still occur due to the non-linearity of the demand. A better option seems to attain the inventory parameters using simulation, this way we account more for the non-linearity of demand.

We showed in this study that current traditional production systems have many shortcomings when dealing with non-linear demand. Moreover, we demonstrated that both parametric and non-parametric forecasting methods lead to stock-outs when the demand was non-linear. The limitations of the non-parametric methods, and in particular, the exponential smoothing one, were described and proved by Croston [29]. As for the parametric forecasting, previous works in the literature have showed the inappropriateness of the Normality assumption, and recommended that the correct \( D_L \) distribution be used when estimating the inventory parameters.

The simulation results also showed that parameters generated following the correct \( D_L \) lead to stock-outs such that the demands were either overestimated or under estimated. Highly erratic demand which sometime fit normal distribution is not adequately controlled using a normal distribution as shown in the normal approximation, stock-outs would still occur due to the non-linearity of the demand. A better option seems to attain the inventory parameters using simulation, this way we account more for the non-linearity of demand.

We showed in this study that current traditional production systems have many shortcomings when dealing with non-linear demand. Moreover, we demonstrated that both parametric and non-parametric forecasting methods lead to stock-outs when the demand was non-linear. The limitations of the non-parametric methods, and in particular, the exponential smoothing one, were described and proved by Croston [29]. As for the parametric forecasting, previous works in the literature have showed the inappropriateness of the Normality assumption, and recommended that the correct \( D_L \) distribution be used when estimating the inventory parameters.

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<table>
<thead>
<tr>
<th>Optimal CSL result of distribution parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part-type 1</td>
</tr>
<tr>
<td>Part-type 2</td>
</tr>
<tr>
<td>Part-type 3</td>
</tr>
<tr>
<td>Part-type 4</td>
</tr>
<tr>
<td>Part-type 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulation result of distribution parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal ROP</td>
</tr>
<tr>
<td>Part-type 1</td>
</tr>
<tr>
<td>Part-type 2</td>
</tr>
<tr>
<td>Part-type 3</td>
</tr>
<tr>
<td>Part-type 4</td>
</tr>
<tr>
<td>Part-type 5</td>
</tr>
</tbody>
</table>
results. A better approach is to calculate the order point directly from real demand where applicable (customer orders). In doing so accurately determine the degree of safety stock required through simulation.

In general, forecasting techniques can generate optimal results that are relatively good depending on how good the inventory parameters are. Although stock-outs would still occur, due to the demand non-linearity of the demand when demand data with fitted distributions were examined, however, it was shown that fitting distribution to data is relatively better when compared with the normal approximations. Simulation technique was found to be a superior approach to predict the best inventory parameters, especially particularly in manufacturing systems with non-linear constraints.

It was shown that current traditional manufacturing systems have numerous shortcomings when dealing with non-linear demand. For instance, Material Requirement Planning (MRP) uses forecasting techniques, and, due to the complexity of non-linear constraint, fails to response to surge or variation in non-linear constraints. On the hand, pull control strategies, such as Kanban systems, use lot sizes that are not flexible for high production volume and mix variations, and as such unresponsive to erratic and intermittent demand.

In both parametric and non-parametric forecasting techniques, stock-outs occur in multi-product manufacturing systems under erratic or non-linear constraints. The limitations of the non-parametric methods, especially the exponential smoothing techniques were shown to have an adverse effect on the performance of a system. Similarly the parametric forecasting, especially the inappropriateness of the normality assumption, was shown. It was recommended that the correct distribution be used, when estimating the inventory parameters.

It was also shown in Table 7 that the inventory parameters generated using the appropriate distribution as recommended from the goodness of fit test resulted in stock-outs. The outcome of the distributions used turned out to either overestimate or under estimate the demand. A better approach is to calculate the order point directly from actual demand where applicable. This process computes near to accuracy the level of safety stock required via the simulation experiments without recourse to the best fitted statistical distribution.

Reference

Evaluation of Forecasting and Inventory Control in Multi-product Manufacturing Systems Operating under Erratic Demand: A Case Study in the Automotive Domain


